

## **Bayesian Inversion of Seabed Scattering Data (Special Research Award in Ocean Acoustics)**

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### **LONG-TERM GOALS**

The long-term goals of this work are to improve ocean acoustic reverberation modeling and sonar performance predictions in shallow waters by developing inversion procedures to estimate seabed geoacoustic and scattering properties and their uncertainties, as well as investigating the importance of various scattering processes. Important issues include: investigating the angular and frequency dependence of scattering (defining the scattering kernel), determining the dependence of scattering on physical properties of the seabed, and establishing the relative importance between scattering due to rough boundaries at the seafloor and sub-bottom interfaces or at volume inhomogeneities. These issues are all key to the ability to invert scattering and/or reflection data for seabed geoacoustic and scattering parameters, and ultimately to the practical inversion of active source reverberation data for rapid environmental assessment applications.

### **OBJECTIVES**

The specific objectives of this project are to carry out Bayesian inversion of scattering and reflection data to estimate seabed scattering and geoacoustic parameters and their uncertainties, and to investigate the relative importance of surface and volume scattering mechanisms in measured data. This involves the development and implementation of efficient forward modeling and trans-dimensional Bayesian

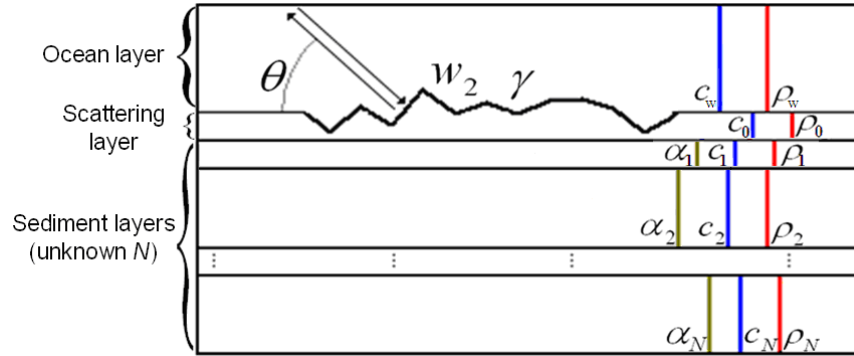
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inversion approaches, the application of these approaches to measured data, and interpretation of results.

## APPROACH

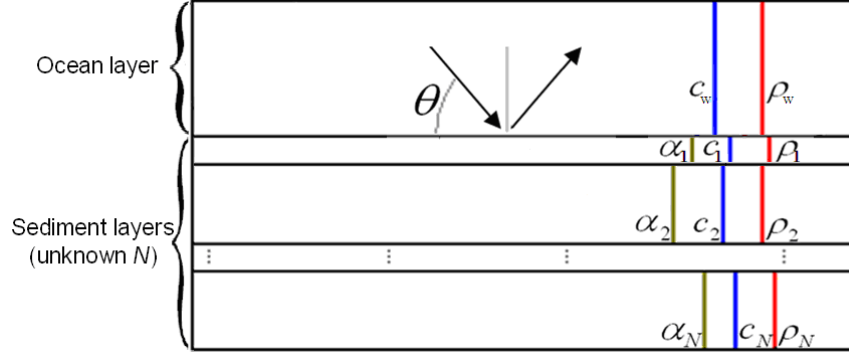
The data used in this work were collected by Charles Holland, who measured direct-path scattering<sup>1</sup> and reflection<sup>2</sup> data over a wide frequency band at a number of shallow-water test beds (Malta Plateau and near Elba Island in the Mediterranean Sea, and New Jersey and Scotian Shelves in the North Atlantic). These measurements probe the seabed on an intermediate spatial scale (patch-size radius  $\sim 500$  m for both reflection and scattering), which generally allows a laterally-invariant parameterization of the seabed and reduces effects of spatial and temporal variability in the water column and other experimental uncertainties. While such data have been interpreted based on forward modeling studies and supporting geoacoustic measurements<sup>2</sup>, rigorous scattering inversion algorithms have not been developed, nor have quantitative inversion studies been carried out.

The two primary forward models required for this work involve the monostatic scattering kernel and spherical-wave reflection coefficients, both applied to an arbitrary layered seabed. The initial scattering kernel considered here defines the monostatic backscatter of sound from a single rough interface between two fluid layers<sup>3</sup>. Other scattering mechanisms (e.g., a sub-surface scattering interface, multiple scattering interfaces, volume scattering) will also be applied in this research, but have not yet been explored. The surface scattering model is illustrated in Fig. 1. The ocean layer is assumed to be homogenous. The first seabed layer represents the scattering layer, which is underlain by a series of flat sediment layers, terminated by a halfspace ( $N+1$  layers in all). Layer properties include thickness  $h$ , sound speed  $c$ , density  $\rho$ , and attenuation  $\alpha$ . The parameters  $\gamma$  and  $w_2$  represent the scattering exponent and scattering strength, and are used to approximate the 2D roughness power spectrum of the seabed.



**Figure 1: Schematic diagram of the monostatic scattering kernel forward model.**

The second forward model required for this work is the spherical reflection coefficient for an arbitrary  $N$ -layer fluid half-space<sup>4</sup>. Figure 2 shows the assumed sediment structure for the spherical reflection model. An important point is that since the scattering layer in Fig. 1 is assumed to have negligible thickness, it is omitted in Fig. 2.



**Figure 2: Schematic diagram of the spherical reflection coefficient forward model.**

A Bayesian framework is applied for inversions in this work as it provides rigorous estimation of parameter uncertainties as well as parameter estimates, thereby quantifying the information content of the data to resolve the model parameters. Bayesian inversion is based on formulating the posterior probability density (PPD) of the model parameters of interest, which combines both data and prior information<sup>5,6</sup>. The multi-dimensional PPD is interpreted in terms of parameter estimates (e.g., the most-probable model), parameter uncertainties (variances, marginal distributions, credibility intervals), and parameter inter-relationships (correlations, joint marginal distributions), which provide a complete solution to the inverse problem. Since scattering and reflection inversion are strongly non-linear problems, these PPD properties will be computed numerically using global optimization and Markov-chain Monte Carlo (MCMC) sampling algorithms.

One of the goals of this work is to carry out inversions trans-dimensionally, a relatively new approach in ocean acoustic inversion<sup>5</sup>. Trans-dimensional inversion allows the natural parsimony of the Bayesian approach to determine the complexity (e.g., number of seabed layers) of the estimated models. It also allows the parameter uncertainties to be marginalized over the number of layers, hence accounting for the uncertainty of the parameterization within parameter uncertainty estimates. Trans-dimensional inversion has been shown to model complex layering structures without over-parameterizing simple structures<sup>7</sup>. However, trans-dimensional inversion is computationally intensive, and of particular interest is developing adaptive techniques for increasing the efficiency.

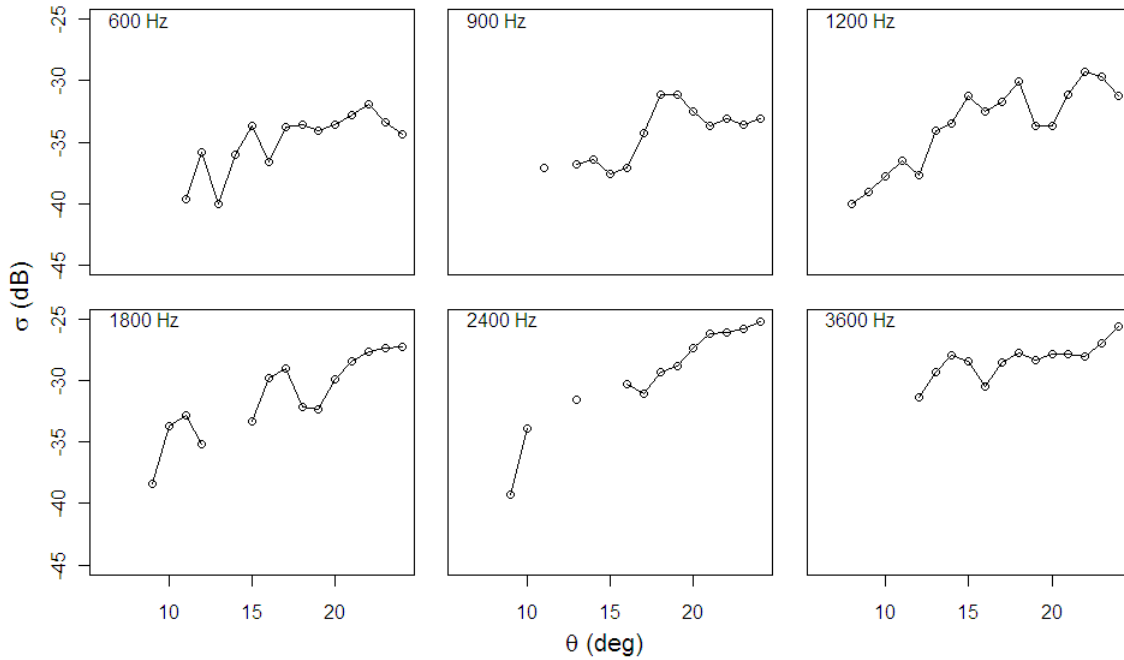
The inversion of the two types of data (reflection and scattering) will be conducted both individually and jointly. The ability to resolve scattering parameters (scattering strength, spatial roughness power spectrum) and geoacoustic parameters (sound speed, density, attenuation, and layering structure) via inversion will be investigated in terms of posterior parameter uncertainty distributions, which quantify the effective data information content. In particular, scattering/geoacoustic parameter resolution will be investigated for inversion of scattering data alone versus joint inversion of scattering and reflection data (joint inversion is expected to overcome inter-parameter correlations which limit parameter resolution in scattering inversion<sup>8</sup>). Of interest is determining the information required to distinguish between different scattering mechanisms and scattering theory approximations; this will be investigated using both measured and simulated data.

## WORK COMPLETED

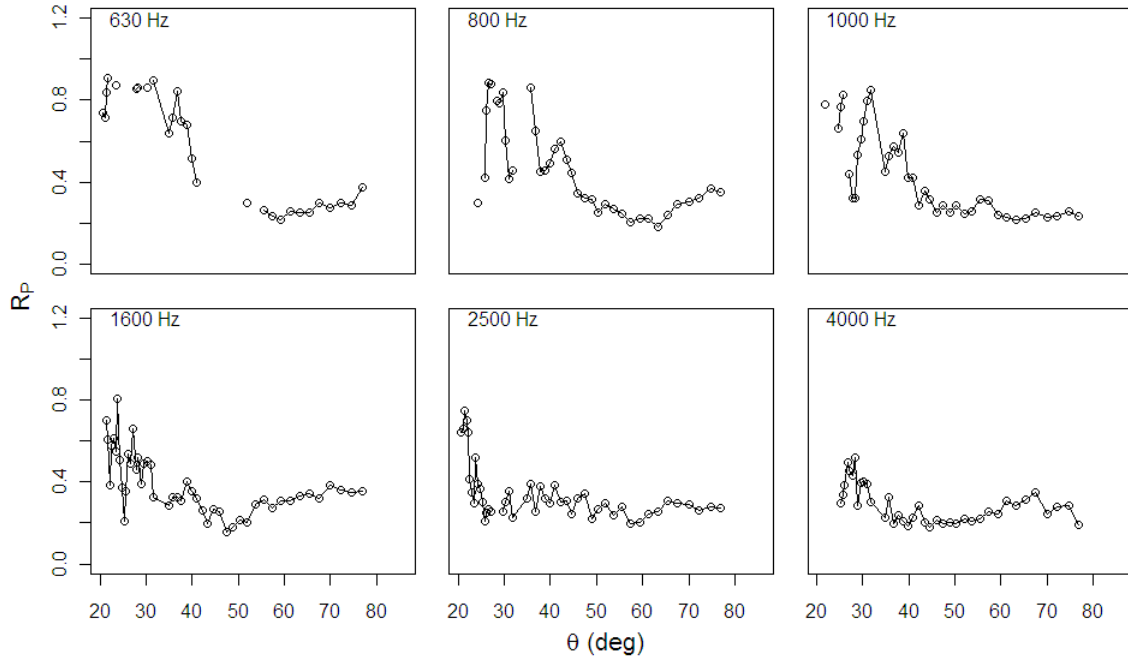
- 1) A fixed-dimensional inversion of the measured scattering data was conducted; the results have been presented at the 2011 meeting of Acoustic Society of America<sup>9</sup>. The inversion assumed only one sediment layer. Geoacoustic parameter estimates in the sound layer were found to be unresolved (their PPDs were approximately uniform). An informative prior distribution constraining the relationship between sediment sound speed and density was developed based on a large collection of empirical measurements reported by Hamilton<sup>10</sup>. Geoacoustic parameter estimates were found to be adequately resolved (their PPDs had a single clear peak) when the Hamilton prior was applied in the inversion. The scattering parameters were well resolved in both cases, with estimated values consistent with expectations for the site and reasonably small uncertainties.
- 2) A joint trans-dimensional scattering/reflection inversion algorithm has recently been developed and applied to measured data. The results are presented in more detail in the following section. This inversion has been conducted with and without the Hamilton prior. Only results for inversion without the Hamilton prior are presented here.

## RESULTS

This section presents the joint trans-dimensional scattering and reflection inversion results. The scattering data (Fig. 3) are at frequencies of 600, 900, 1200, 1800, 2400 and 3600 Hz and an angular range of 5–24°. The reflection data (Fig. 4) have a similar frequency range (630, 800, 1000, 1600, 2500 and 4000 Hz) and an angular range of 20–85°.

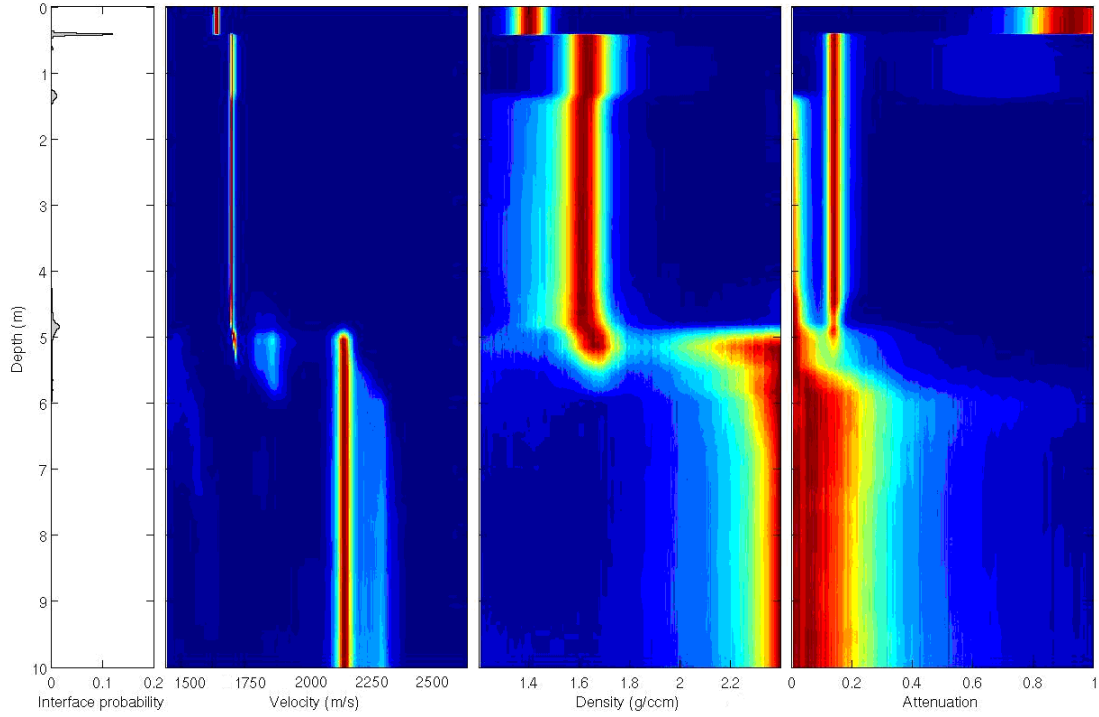


**Figure 3: The measured scattering data as a function of angle and frequency.**

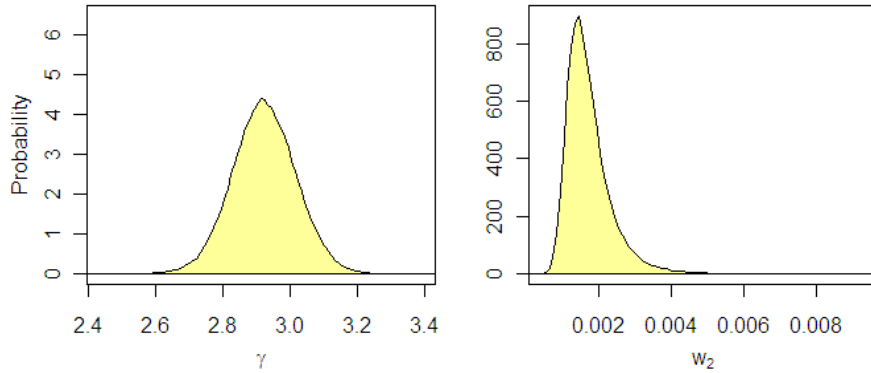


**Figure 4: The measured reflection data as a function of angle and frequency.**

A joint scatter/reflection inversion has been conducted. Due to the high computational requirements of calculating spherical reflection coefficients for high-frequency data, and the fact that spherical-wave effects are reduced at higher frequencies, the forward model for the reflection data was approximated with a plane-wave reflection coefficient at 2500 and 4000 Hz data (spherical-wave reflection modeling was applied at 630, 800, 1000, and 1600 Hz). The marginal posterior profiles of the geoacoustic parameters are displayed in Fig. 5. Overall, 3-5 sediment layers are resolved/supported by the data. The sound speed profile appears better constrained within its prior bounds than the density and attenuation profiles. The sound speed and density of the uppermost layer is consistent with sand (known to represent the surficial sediments). The high sound speed and density below 5-m depth is likely evidence of a limestone basement, which is known to exist in the region. If so, to model the seabed correctly requires extending the forward model to include shear waves in the basement, which will be considered in future work. The marginal probability distributions of the scattering parameters are shown in Fig. 6. These distributions indicate that  $\gamma$  and  $w_2$  are quite well resolved with reasonable estimated values. Additionally the shapes of the distributions appear to be approximately Gaussian and log-Gaussian, respectively (their theoretical distributions if all other parameters are known).



**Figure 5: Marginal posterior profiles for geoaoustic properties from joint scattering/reflection inversion. Plot boundaries are set equal to prior bounds.**



**Figure 6: Marginal posterior probability distributions of the scattering parameters (scaled so each distribution has the same area).**

## IMPACT/APPLICATIONS

The performance of naval sonar systems in shallow waters is strongly influenced by acoustic interaction with the bottom, and therefore knowledge of seabed scattering and geoaoustic parameters and their uncertainties is required to predict and optimize sonar performance. Bayesian inversion methods offer a powerful framework for parameter extraction and uncertainty estimation, thereby quantifying the geoaoustic information content of the data. The proposed inversion methodology has been applied to reflection data, and the current project aims to combine this with monostatic scattering inversion to constrain seabed scattering parameters and investigate scattering mechanisms.

## RELATED PROJECTS

- 1) *Bayesian Seismo-Acoustic Inversion to Investigate Spatial Variability and Uncertainty of Shallow Water Sediments, 2007-2008 (Award Number: N000140710540)*. This project developed software tools that implement Bayesian inversions for sediment sound speed, density and attenuations from reflection-coefficient data similar to that inverted here.
- 2) *Quantifying Geoacoustic Uncertainties and Seabed Variability for Propagation Uncertainty (Award Number: N000140910394)*. This project involves Bayesian inversion of reflection data to investigate the uncertainty and variability of seabed geoacoustic parameters, and the effects on acoustic propagation. Inversion methodologies similar to that in the present project are developed and applied.
- 3) *Bayesian Ambient Noise Inversion for Geoacoustic Uncertainty Estimation (Award Number: N000141110214)*. This project involves trans-dimensional Bayesian inversion of oceanic ambient noise for seabed geoacoustic parameters. The ambient noise is processed to produce bottom-loss as a function of angle and frequency, which is similar to the reflection data inverted in this project. Similar trans-dimensional inversion procedures are also applied.

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